AN INVESTIGATION OF ISI AND ICI PROPERTIES FOR A FAMILY OF OFFSET RAMP IMPROVED NYQUIST FILTERS

Ligia Alexandra ONOFREI, Nicolae Dumitru ALEXANDRU
"Ştefan cel Mare" University of Suceava, str. Universităţii no.13, RO-720225, Suceava, onofreial@eed.usv.ro,
"Gh. Asachi" Technical University of Iaşi, , Bd. Carol I, no 11, RO-700506 Iaşi, nalex@etc.tuiasi.ro

Abstract: We proposed a family of new Nyquist pulses based on an offset ramp method and investigated its performances in terms of ISI error probability. These pulses proposed and studied are employed for OFDM use, to reduce the sensitivity of OFDM systems to frequency offset. The results presented in this paper equal or outperform the recently found pulses in terms of intercarrier interference (ICI) power.
Key words: *intersimbol-interference (ISI), intercarrier-interference (ICI), frequency offset*

I. INTRODUCTION

We propose a family of new Nyquist pulses that shows comparable or better ISI performance in the presence of sampling errors, as compared with some recently proposed pulses.

In this paper we evaluate the performance of new *ISI*-free pulses in transmission on the OFDM systems. The performances are studied with respect to the ISI error probability and inter-carrier interference (ICI) in OFDM systems.

II. THE NEW PULSE CHARACTERISTICS

Recent works have reported and examined new families of pulses which are intersymbol interference (*ISI*)- free. These pulses are produced by a Nyquist lowpass filter with odd symetry around the ideal cutoff frequency.

In this paper we propose a new Nyquist pulse shape which is designed using the *flipped*-$G(f)$ technique [2, 3, 4].

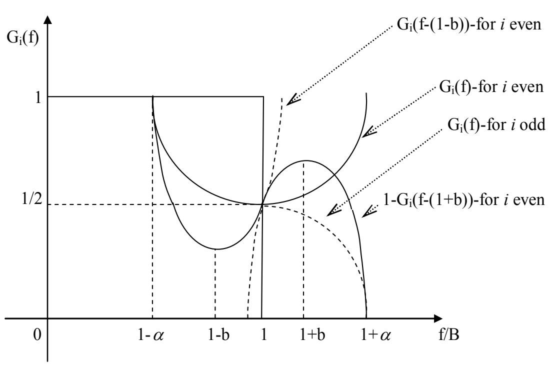


Figure 1. The construction technique used for the new pulse

The characteristic of the new family of pulses is defined using equation (1) and is illustrated in figure 2.

$$G_i(f) = d\left(f - (1-b)\right)^i + c \qquad (1)$$

where: b is the offset of the ramp characteristic with respect to Nyquist frequency

c, d are constants, which were chosen in order for the function $G(f)$ to satisfy:

$$G_i(1) = 1/2 \text{ şi } G_i(1-\alpha) = 1 \qquad (2)$$

and α represents the excess bandwidth;

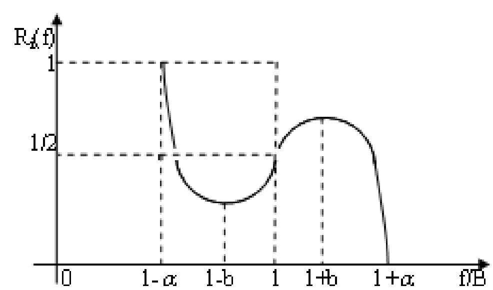


Figure 2. Proposed filter frequency characteristic

$G(f)$ was chosen to have a concave shape in the frequency interval $B(1-\alpha) \le |f| \le B$ in order to transfer some energy to the high frequency spectral range [2], [3], [4].

B is the bandwidth corresponding to symbol repetition rate $T = 1/2B$.

For i - even, $G(f)$ shows even symmetry around $(1-b)B$ and the filter characteristic $R_i(f)$ is defined for positive frequencies as:

$$R_i(f)=\begin{cases}1 & , \quad |f|\le B(1-\alpha)\\ G_i(f-(1-b)) & , \quad B(1-\alpha)\le|f|\le B\\ 1-G_i(f-(1+b)) & , \quad B\le|f|\le B(1+\alpha)\\ 0 & , \quad inrest\end{cases} \tag{3}$$

For i - odd they show odd symmetry around $(1-b)B$ and the $R_i(f)$ definition is:

$$R_i(f)=\begin{cases}1 & , \quad |f|\le(1-\alpha)\\ 2c-G_i(f-(1-b)) & , \quad B(1-\alpha)\le|f|\le B(1-b)\\ G_i(f-(1-b)) & , \quad B(1-b)\le|f|\le B\\ 1-G_i(f-(1+b)) & , \quad B\le|f|\le B(1+b)\\ 1-2c+G_i(f-(1+b)) & , \quad B(1+b)\le|f|\le B(1+\alpha)\\ 0 & , \quad inrest\end{cases} \tag{4}$$

Figures 3, 4 and 5 illustrate the frequency and time characteristics of the proposed family of pulses (*i=2, 3* and *4*) for different values of the excess bandwidth.

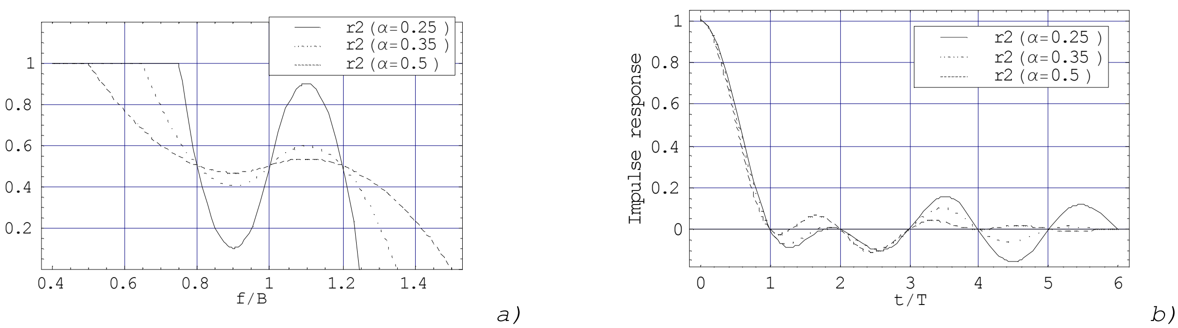


Figure 3. a) Frequency characteristics of the proposed pulses; b) Impulse responses for several roll-off factors α and i=2, b=0.1;

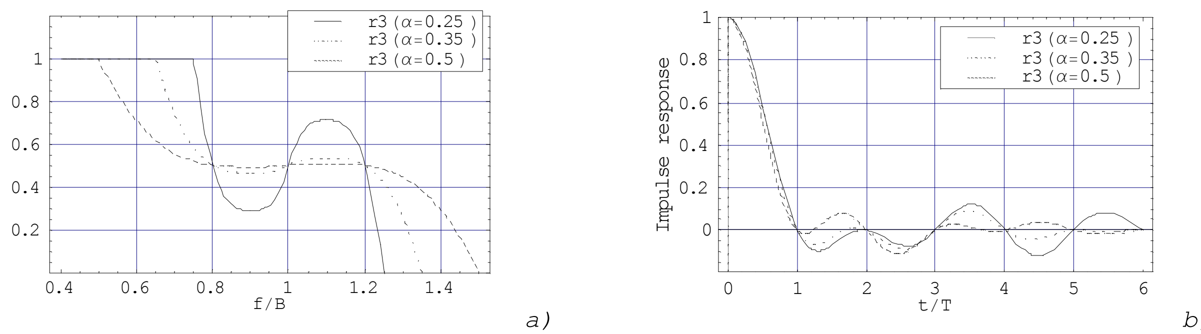


Figure 4. a) Frequency characteristics of the proposed pulses; b) Impulse responses for several roll-off factors α and i=3, b=0.1;

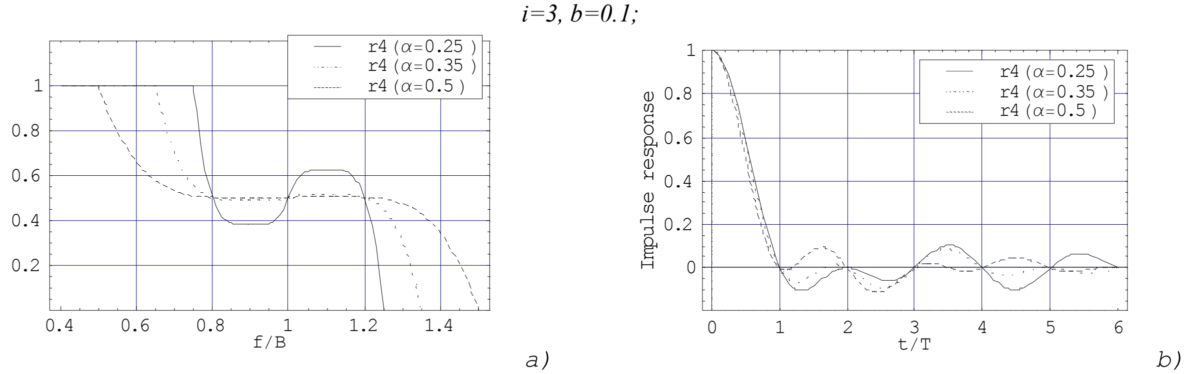


Figure 5. a) Frequency characteristics of the proposed pulses; b) Impulse responses for several roll-off factors α and i=4, b=0.1;

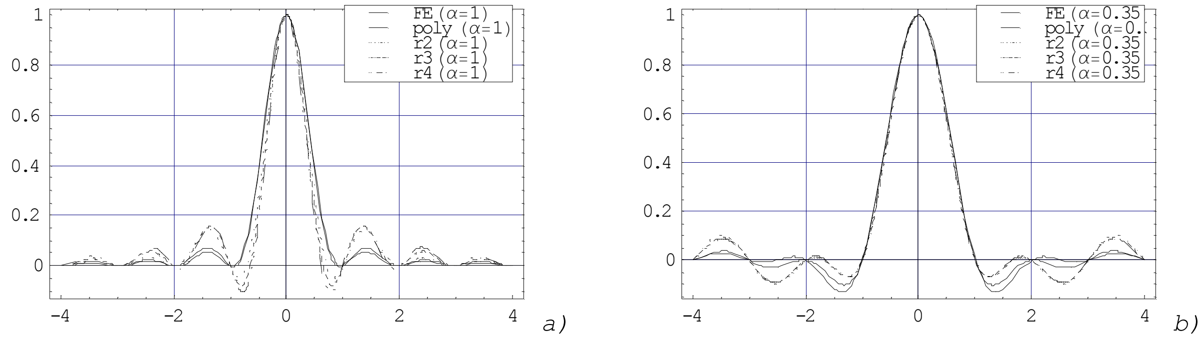


Figure 6. Impulse responses of the proposed pulses a) $\alpha = 1$; b) $\alpha = 0.35$;

Figure 6 shows the impulse response of this family of new Nyquist filter characteristics (*i=2,3* and *4*) together with the flipped exponential (FE) defined in [2] and the polynomial pulse (poly) defined in [20] taken as a reference. We observe, from the figure 6, that although the decrease of the first side lobe is more significant, the next side lobes are significantly larger, which results in increased ISI.

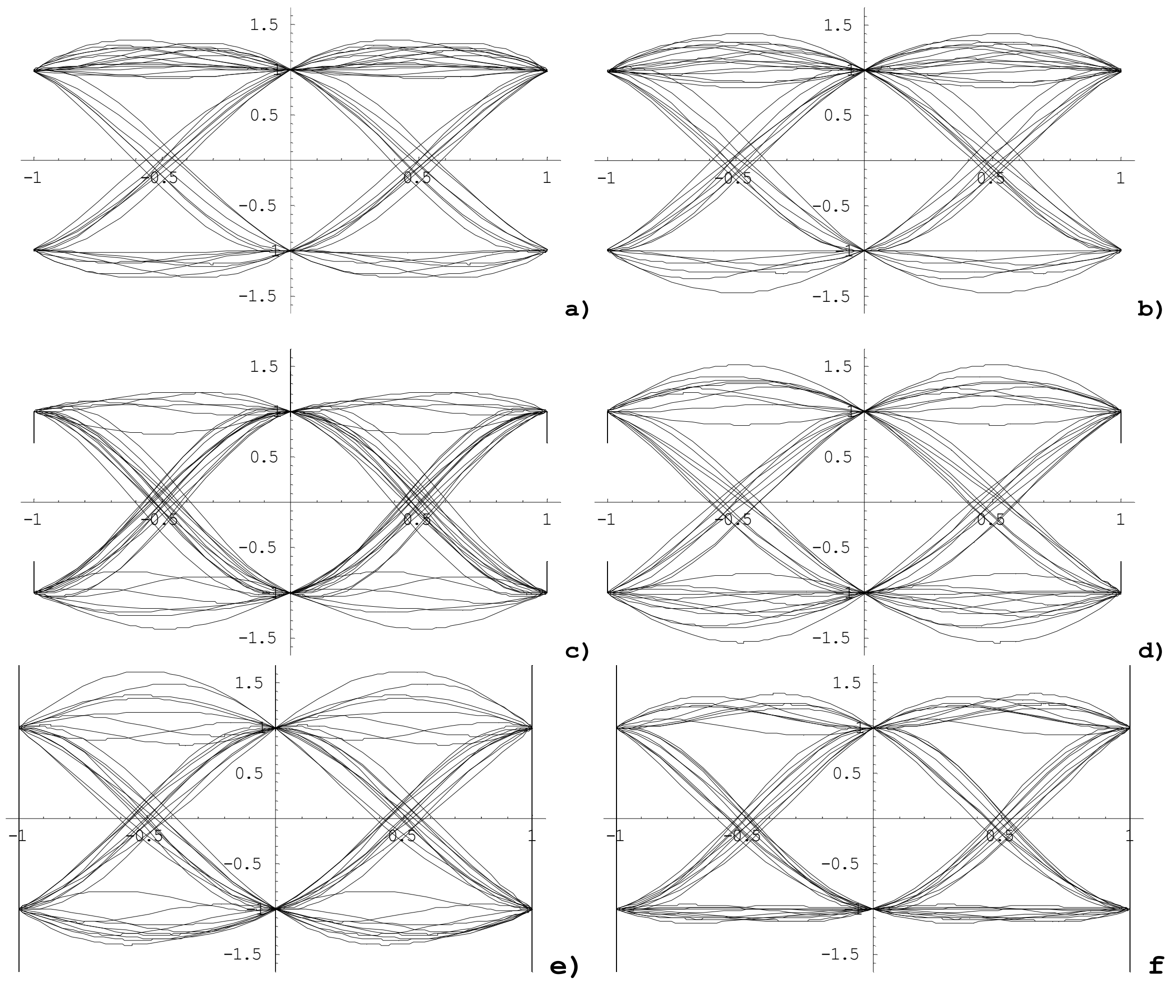


Figure 7. Eye diagram of the proposed pulses
a) i=2, $\alpha = 0.35$; b) i=2, $\alpha = 1$; c) i=3, $\alpha = 0.35$; d) i=3, $\alpha = 1$; e) i=4, $\alpha = 0.35$; f) i=4, $\alpha = 1$;

Table 1. ISI error probability of the proposed Nyquist pulses for N=2^{10} interfering symbols and SNR = 15 dB

P_e		t/T=0.05		t/T=0.1		t/T=0.2	
α	b	b=0.01	b=0.05	b=0.01	b=0.05	b=0.01	b=0.05
0.25	**FE**	**5.812*10^{-8}**		**1.298*10^{-6}**		**3.568*10^{-4}**	
	r2	5.25147*10^{-8}	5.1552*10^{-8}	1.07079*10^{-6}	1.0845*10^{-6}	2.81911*10^{-4}	3.00334*10^{-4}
	r3	5.01381*10^{-8}	4.97884*10^{-8}	1.00399*10^{-6}	1.02289*10^{-6}	2.67107*10^{-4}	2.81978*10^{-4}
	r4	4.92714*10^{-8}	4.91255*10^{-8}	9.95403*10^{-7}	1.0149*10^{-6}	2.70945*10^{-4}	2.84057*10^{-4}
	poly	4.734*10^{-8}		8.834*10^{-7}		2.241*10^{-4}	
0.35	**FE**	**3.925*10^{-8}**		**5.402*10^{-7}**		**1.013*10^{-4}**	
	r2	3.5378*10^{-8}	3.50306*10^{-8}	4.58546*10^{-7}	4.72939*10^{-7}	8.71313*10^{-5}	9.75018*10^{-5}
	r3	3.40248*10^{-8}	3.40444*10^{-8}	4.52313*10^{-7}	4.69101*10^{-7}	9.38418*10^{-5}	1.03171*10^{-4}
	r4	3.37666*10^{-8}	3.39011*10^{-8}	4.68468*10^{-7}	4.85833*10^{-7}	1.05149*10^{-4}	1.14328*10^{-4}
	poly	3.290*10^{-8}		3.839*10^{-7}		6.563*10^{-5}	
0.5	**FE**	**2.413*10^{-8}**		**1.858*10^{-7}**		**2.088*10^{-5}**	
	r2	2.20319*10^{-8}	2.20699*10^{-8}	1.67002*10^{-7}	1.73685*10^{-7}	2.27583*10^{-5}	2.64656*10^{-5}
	r3	2.17483*10^{-8}	2.19696*10^{-8}	1.73602*10^{-7}	1.80743*10^{-7}	3.01763*10^{-5}	3.41075*10^{-5}
	r4	2.21378*10^{-8}	2.24301*10^{-8}	1.80743*10^{-7}	1.9493*10^{-7}	3.90994*10^{-5}	4.3683*10^{-5}
	poly	2.057*10^{-8}		1.354*10^{-7}		1.520*10^{-5}	

III. SIMULATION RESULTS

A. NUMERICAL RESULTS FOR ERROR PROBABILITY

Figure 7 illustrates the receiver eye diagram for new pulses and the effects of the ICI is estimated in the following. When the receiver eye diagram is sampled off center, as in practical receivers, timing error results in an increase of the average symbol error probability [2, 3]. The new families of Nyquist pulses show reduced maximum distortion, a more open receiver eye and decreased symbol error probability [4] in the presence of timing error, as compared with the *flipped-exponential* (FE) pulse [2] with the same roll-off factor.

Error probability P_e can be evaluated as in [12] using Fourier series.

$$P_e = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ M\,odd}}^{M} \left(\frac{\exp\left(-m^2\omega^2/2\right)\sin\left(m\omega g_0\right)}{m} \right) \prod_{k=N_1}^{N_2} \cos\left(m\omega g_k\right)$$

M represents the number of coefficients considered in the approximate Fourier series of noise complementary distribution function; $\omega = \frac{2\pi}{T_f}$ angular frequency; T_f is the period used in the series; N1 and N2 represent the number of interfering symbols before and after the transmitted symbol; $g_k = p(t-kT)$ where p(t) is the pulse shape used and T is the bit interval.

The ISI error probability is calculated using the method of [12]] in the presence of time sampling errors for all proposed pulses and is illustrated in Table 1, together with those for FE pulse and polynomial pulse. The results are computed using $T_f = 40$ and $M = 61$ for N=2^{10} interfering symbols and SNR = 15dB.

From the results listed in Table 1. we observe that the new pulses outperform the *flipped-exponential* (FE) pulse and are slightly worse than polynomial pulse. As expected, the error probability is reduced if the roll-off factor is increased.

The error probability measure the performances of the pulses regarding intersymbol intereferences and includes the effects of noise, synchronization error and distotion.

B. OFDM USE

The proposed and studied pulses can also be used for reducing average *ICI* power in OFDM systems [19]. In the sequel we followed the same model as in [19] in order to evaluate the average *ICI* power and the average signal power to average *ICI* power ratio (*SIR*). The simulations results are obtained for the 64-subcarrier OFDM systems.

In figure 8 it is plotted the average ICI power for the pulses studied together with *flipped-exponential* pulse and polynomial pulse taken as references. The ICI power for the new pulses is smaller than ICI power for *flipped-exponential* pulse. The results are for the case when $\alpha = 0.35$ and b=0.01.

The increase of the roll-off factor α is expected to result in the reduction of ICI power. An increasing of α corresponds to reducing the side lobes in the spectrum. From the figure 8a) and 8b) we observe that the studied pulses show better results than *flipped-exponential* pulse and similar results with polynomial pulses [20].

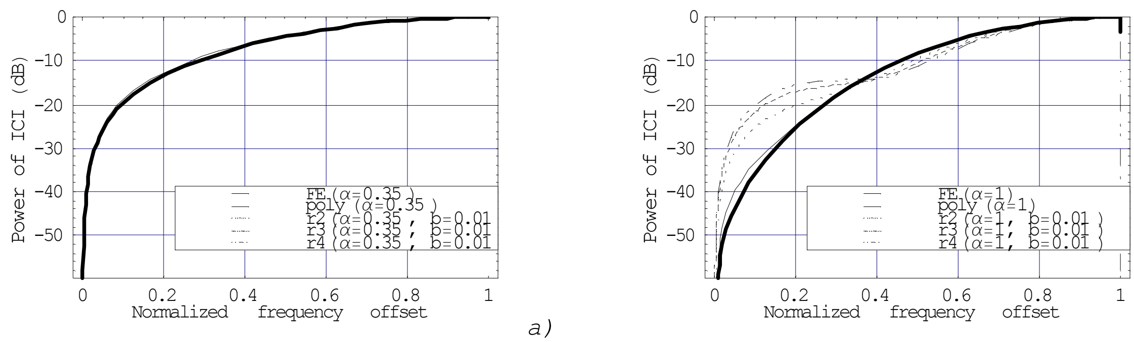
As shown in figure 8b), which presents the ICI power for roll-off factor $\alpha = 1$ we observe that the behavior of the pulses is different from that of pulses taken as a reference. When normalized frequency offset is bigger than 0.35, the proposed pulses outperform the references pulses in terms of average ICI power.

For the proposed pulses the best results regarding the average ICI power are obtained when *i=4*. The results are plotted in the figure 8 and figure 9.

If $\Delta fT = 0.49$ and $b = 0.1$ then average ICI power is -10.1492dB, -11.1467dB and -11.9107dB for *i=2, i=3* and *i=4* respectively. When *i=4*. the new pulse achieves 3.13002dB and 2.86956dB smaller ICI power than the *flipped-exponential* pulse for *b=0.01* and *b=0.1*, respectively. Comparing with the polynomial pulse the new pulse achieves 3.02904dB and 3.28949dB smaller ICI power for *b=0.01* and *b=0.1*, respectively.

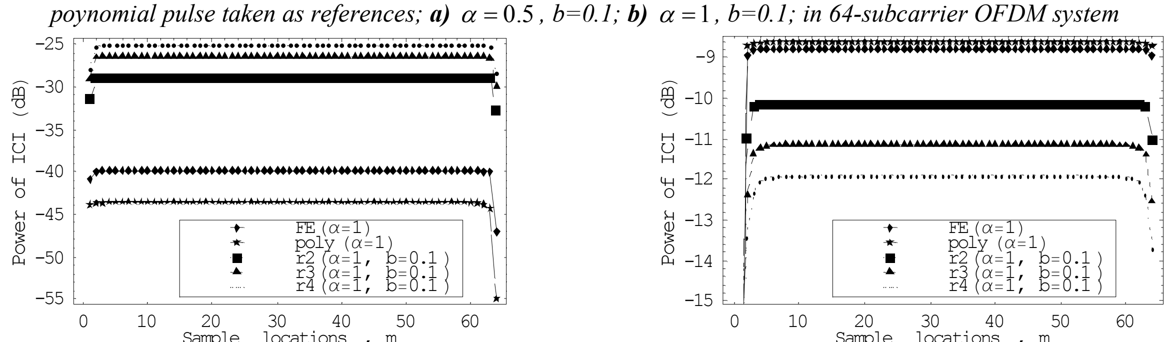
The variations of average ICI power with sample location *m* are presented in figures 9.a) and 9.b) for normalized frequency offset $\Delta fT = 0.05$ and $\Delta fT = 0.49$, respectively. The roll-off factor is $\alpha = 1$ and b=0.01. Comparing the figure 9.a) with 9.b) we observe the improvement of the reductions of ICI power due to the increasing of the roll-off factor α .

As expected, the *ICI* drops for the samples located near sample locations 0 and *N-1*, because these samples have fewer interfering samples. The proposed pulses perform better than considered *flipped-exponential* pulse, as seen in the figure 8. When we concern on the comparative performance of these pulses in terms of the average signal power to average *ICI* power ratio denoted as *SIR* in 64-subcarrier OFDM system [19], the results are plotted in figure 10. a) and figure 10. b).



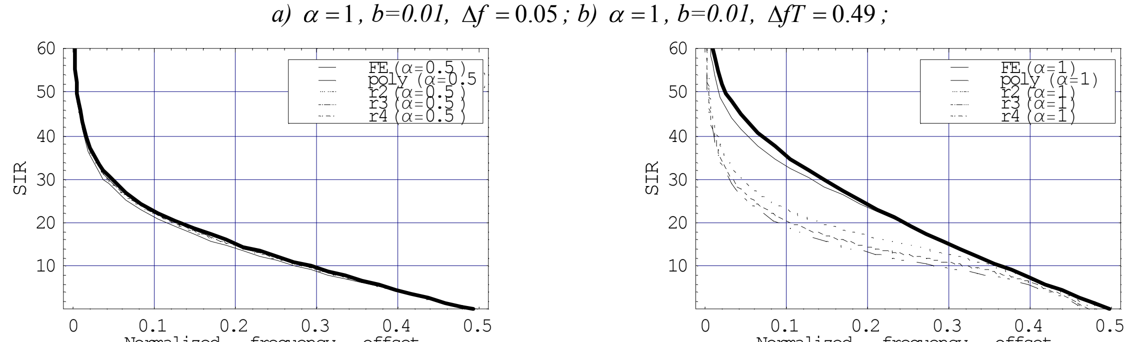
a) b)

Figure 8. The ICI power for the new pulses proposed ($R_i(f)$) together with the flipped-exponential pulse and poynomial pulse taken as references; ***a)*** *$\alpha = 0.5$, b=0.1;* ***b)*** *$\alpha = 1$, b=0.1; in 64-subcarrier OFDM system*



a) b)

Figure 9. The ICI power for different sample location in 64-subcarrier OFDM system
a) $\alpha = 1$, b=0.01, $\Delta f = 0.05$; b) $\alpha = 1$, b=0.01, $\Delta fT = 0.49$;



a) b)

Figure 10. The SIR for the new pulses proposed ($R_i(f)$) together with the flipped-exponential pulse and poynomial pulse taken as references; a) $\alpha = 0.5$, b=0.01; b) $\alpha = 1$, b=0.01; in 64-subcarrier OFDM system

IV. LINEAR COMBINATION OF THE PROPOSED PULSES

In this section we proposed the linear combination [5] between the new pulses studied previously ($R_i(f)$). The linear combination of two pulses guarantees that the resulting pulse has a bandwidth not greater than that of constituent pulse with larger bandwidth, and if the constituent pulses are ISI-free, then the resulting pulse will also be ISI-free [5].

In figure 11 are plotted the linear combinations of the new pulses with the frequency characteristic presented in figure 2.

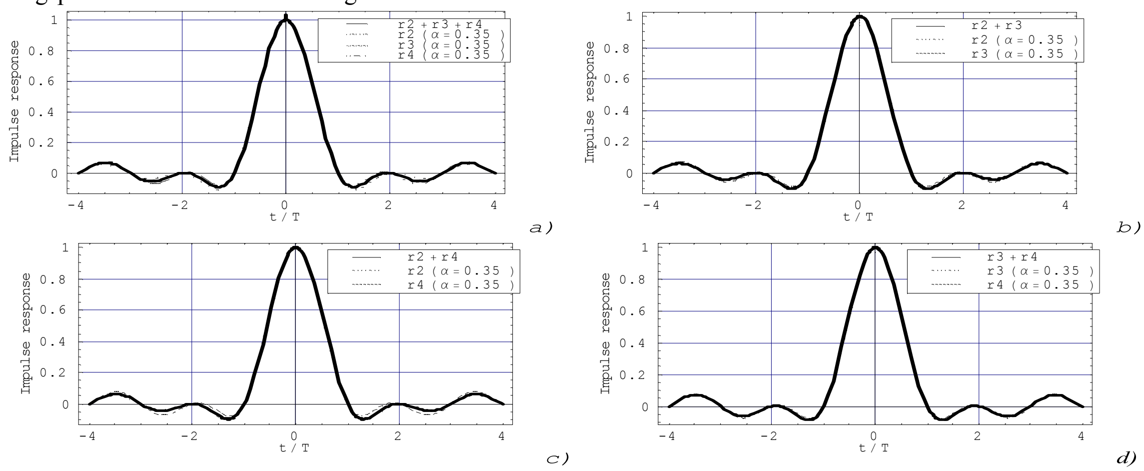


Figure 11. Impulse response of the resulting pulses from linear combinations proposed

The linear combinations for the cases *a, b, c* and *d*, respectively are defined mathematically by:

$$\frac{1}{3}r_2(t)+\frac{1}{3}r_3(t)+\frac{1}{3}r_4(t) \tag{5}$$

$$q*r_2(t)+(1-q)*r_3(t) \tag{6}$$

$$q*r_2(t)+(1-q)*r_4(t) \tag{7}$$

$$q*r_3(t)+(1-q)*r_4(t) \tag{8}$$

The ISI error probability is calculated using the method of [12] in the presence of time sampling errors for all proposed pulses and illustrated in Table 2 (Appendix A), together with those for FE pulse and polynomial pulse. The results are computed using $T_f = 40$ and $M = 61$ for N=2^{10} interfering symbols and SNR=15dB. For diferent values of the parameters α , b and q the resulting pulses show similar behavior with the constituent pulses and outperform the flipped-exponential pulse taken as a reference.

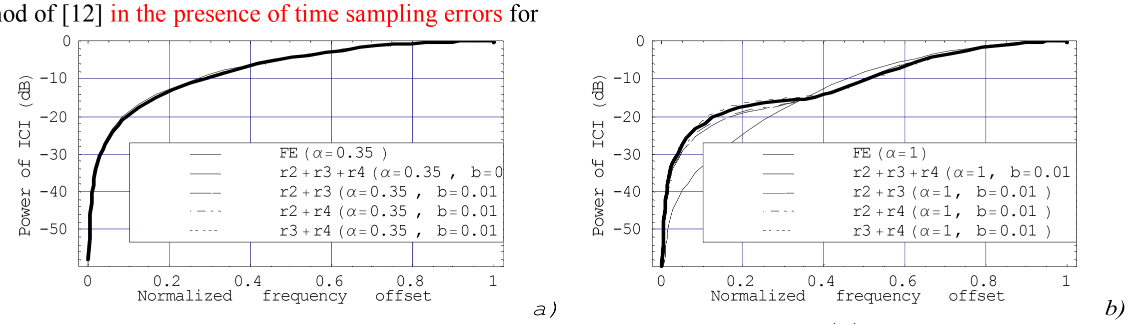


Figure 12. The ICI power for the linear combination of the new pulses proposed ($R_i(f)$) together with the flipped-exponential pulse and polynomial pulse taken as references;a) $\alpha = 0.35$, b=0.01, b) $\alpha = 1$, b=0.01

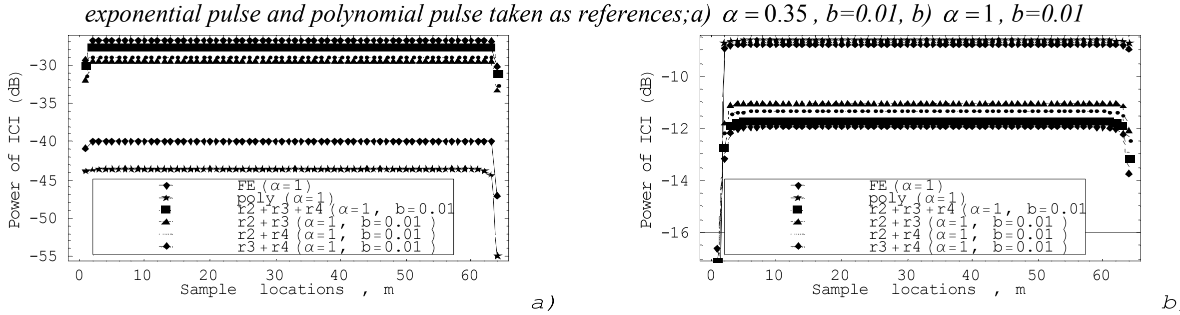


Figure 13. The ICI power for different sample location in a 64-subcarrier OFDM system
a) $\alpha = 1$, b=0.01, $\Delta f = 0.05$; b) $\alpha = 1$, b=0.01, $\Delta f = 0.47$;

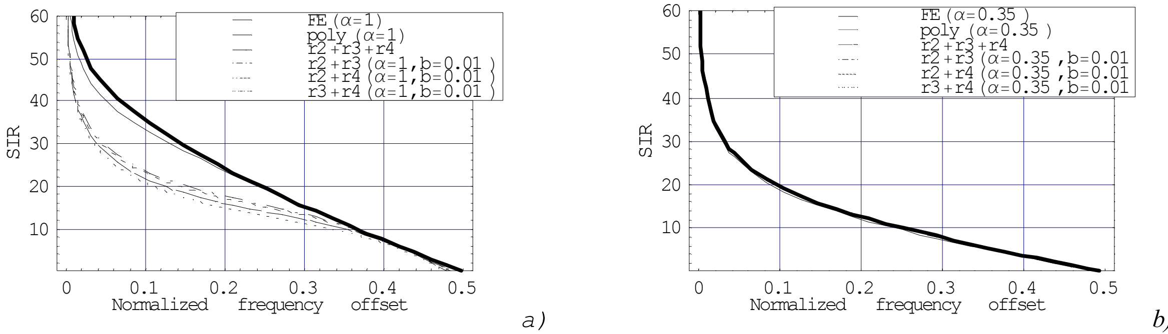


Figure 14. The SIR for the linear combination of the new pulses proposed ($R_i(f)$) together with the flipped-exponential pulse and poynomial pulse taken as references; a) $\alpha = 0.5$, b=0.01; b) $\alpha = 1$, b=0.01; in a 64-subcarrier OFDM system

V. CONCLUSION

We proposed and investigated the performances of a family of new improved Nyquist pulses based on an offset ramp method that show a decreased symbol error probability in the presence of timing error as compared with the FE pulse [2] with the same roll-off factor.

We also presented and evaluated the employment of new *ISI*-free pulses in an OFDM system. The pulses are used for reducing *ICI* in OFDM systems. The results are examined in terms of average *ICI* power and average signal power to average *ICI* power ratio denoted as *SIR*.

The new pulses show improvement in the reduction of *ICI* caused by the frequency offset and appear to be suitable for transmission in the OFDM systems. When normalized frequency offset is bigger than 0.35 the proposed pulses outperform the references pulses in terms of average ICI power.

The calculations of the equations for the error probability, the *ICI* power and the *SIR* were performed using *MATHEMATICA*.

REFERENCES

[1] Nyquist, H.,"Certain topics in telegraph transmission theory" *AIEE Trans.*, vol. 47, pp. 617–644, 1928.

[2] Beaulieu, N. C., Tan, C. C. and Damen, M. O., "A "better than" Nyquist pulse," *IEEE Commun. Lett.*, vol. 5, pp. 367–368, Sept., 2001.

[3] Assalini, A. and Tonello A. M., "Improved Nyquist pulses," *IEEE Communications Letters,* vol. 8, pp. 87 - 89, February 2004.

[4] Beaulieu N. C. and Damen, M. O., "Parametric construction of Nyquist-I pulses," *IEEE Trans. Communications,* vol. COM-52, pp. 2134 - 2142, December 2004.

[5] Sandeep P., Chandan S. and Chaturvedi A. K., (2005) "ISI-Free pulses with reduced sensitivity to timing errors", *IEEE Communications Letters,* vol. 9, no.4, pp. 292 - 294, April.

[6] Demeechai, T., "Pulse-shaping filters with ISI-free matched and unmatched filter properties," *IEEE Trans. Commun.*, vol. 46, p. 992, Aug., 1998.

[7] Tan, C. C. and Beaulieu, N. C., "Transmission properties of conjugate-root pulses," *IEEE Trans. Commun.*, vol. 52, pp. 553–558, Apr. 2004.

[8] Franks, L. E., "Further results on Nyquist's problem in pulse transmission," *IEEE Trans. Commun. Technol.*, vol. COM-16, pp. 337–340, Apr. 1968.

[9] Kisel A. V., "An extension of pulse shaping filters theory," *IEEE Trans. Commun.*, vol. 47, pp. 645–647, May. 1994

[10] Andrisano, O. and Chiani, M., "The first Nyquist criterion applied to coherent receiver design for generalized MSK signals," *IEEE Trans. Commun.*, vol. 42, Feb./Mar./Apr. 1999.

[11] Sayar, B. and Pasupathy, S., "Nyquist 3 pulse shaping in continuous phase modulation," *IEEE Trans. Commun.*, vol. COM-35, pp. 57–67, Jan. 1987.

[12] Beaulieu, N. C., "The evaluation of error probabilities for intersymbol and cochannel interference," *IEEE Trans. Commun.*, vol. 31, pp.1740–1749, Dec. 1991.

[13] Hill Jr., F. S., "A unified approach to pulse design in data transmission," *IEEE Trans. Commun.*, vol. COM-25, pp. 346–354, Mar. 1977.

[14] Xia, X.-G., "A family of pulse-shaping filters with ISI-free matched and unmatched filter properties," *IEEE Trans. Commun.*, vol. 45, pp. 1157–1158,Oct. 1997.

[15] Alexandru, N.D. and Onofrei, L.A., "A Novel Class of Improved Nyquist Pulses", ECUMICT 2006, Ghent, Belgium, pp. 363-370, 2006.

[16] Alexandru, N.D., Alexandru M.L. and Onofrei, L.A., "A Class Of Improved Pulses Generated By Nyquist Filters", Advances In Electrical Engineering, vol 2, pp. 10-14., 2006

[17] Alexandru, N.D. and Onofrei, L.A., "A Class Of ISI-Free And Bandlimited Pulses", DAS 2006, pp 189-194, Suceava, mai 2006.

[18] Alexandru, N.D., and Onofrei, L.A., „A survey of several classes of improved pulses generated by Nyquist filters", Comm'2006, București, 2006.

[19] Tan, P. and Beaulieu, N.C., "Reduced ICI in OFDM System Using the "Better Than" Raised-Cosine Pulse", *IEEE Communications Letters*,vol. 8, no.3, pp.135-137, March 2004.

[20] Chandan S., Sandeep P. and Chaturvedi A. K., "A family of ISI-free polynomial pulses", IEEE Communications Letters, vol. 9, no.6, pp. 496-498, June. 2005

[21] Pollet T., Bladel M.V.,Moeneclaey M., "BER sensitivity of OFDM systems to carrier frequency offset and wiener phase noise" *IEEE Trans. Commun*., vol. 4, pp. 191-193, Feb./Mar./Apr.1999.

APPENDIX:

Table 2. ISI error probability of the liniar combination of the proposed Nyquist pulses for $N=2^{10}$ interfering symbols and SNR = 15 dB

P_e	q	0.7					
	t/T	t/T=0.05		t/T=0.1		t/T=0.2	
α	b	b=0.01	b=0.05	b=0.01	b=0.05	b=0.01	b=0.05
0.25	FE	**$5.812*10^{-8}$**		**$1.298*10^{-6}$**		**$3.568*10^{-4}$**	
	r1+r2+r3	*$5.0121\ 10^{-8}$*	*$4.9832\ 10^{-8}$*	*$9.9596\ 10^{-7}$*	*$1.0228\ 10^{-6}$*	*$2.6251\ 10^{-4}$*	*$2.8150\ 10^{-4}$*
	r2+r3	$5.1554\ 10^{-8}$	$5.0880\ 10^{-8}$	$1.0374\ 10^{-6}$	$1.0579\ 10^{-6}$	$2.7224\ 10^{-4}$	$2.9152\ 10^{-4}$
	r2+r4	$5.0899\ 10^{-8}$	$5.0419\ 10^{-8}$	$1.0136\ 10^{-6}$	$1.0410\ 10^{-6}$	$2.6502\ 10^{-4}$	$2.8626\ 10^{-4}$
	r3+r4	$4.9779\ 10^{-8}$	$4.9519\ 10^{-8}$	$9.9622\ 10^{-7}$	$1.0166\ 10^{-6}$	$2.6620\ 10^{-4}$	$2.8105\ 10^{-4}$
	poly	**$4.734*10^{-8}$**		**$8.834*10^{-7}$**		**$2.241*10^{-4}$**	
0.35	FE	**$3.925*10^{-8}$**		**$5.402*10^{-7}$**		**$1.013*10^{-4}$**	
	r1+r2+r3	*$3.3904\ 10^{-8}$*	*$3.3957\ 10^{-8}$*	*$4.4303\ 10^{-7}$*	*$4.6266\ 10^{-7}$*	*$8.9642\ 10^{-5}$*	*$1.0019\ 10^{-4}$*
	r2+r3	$3.4741\ 10^{-8}$	$3.4571\ 10^{-8}$	$4.4878\ 10^{-7}$	$4.6592\ 10^{-7}$	$8.6560\ 10^{-5}$	$9.7167\ 10^{-5}$
	r2+r4	$3.4288\ 10^{-8}$	$3.4231\ 10^{-8}$	$4.4072\ 10^{-7}$	$4.6024\ 10^{-7}$	$8.5551\ 10^{-5}$	$9.6649\ 10^{-5}$
	r3+r4	$3.3854\ 10^{-8}$	$3.3923\ 10^{-8}$	$4.5388\ 10^{-7}$	$4.7126\ 10^{-7}$	$9.6046\ 10^{-5}$	$1.0543\ 10^{-4}$
	poly	**$3.290*10^{-8}$**		**$3.839*10^{-7}$**		**$6.563*10^{-5}$**	
0.5	FE	**$2.413*10^{-8}$**		**$1.858*10^{-7}$**		**$2.088*10^{-5}$**	
	r1+r2+r3	*$2.1533\ 10^{-8}$*	*$2.1770\ 10^{-8}$*	*$1.6765\ 10^{-7}$*	*$1.7543\ 10^{-7}$*	*$2.7706\ 10^{-5}$*	*$3.1765\ 10^{-5}$*
	r2+r3	$2.1744\ 10^{-8}$	$2.1869\ 10^{-8}$	$1.6536\ 10^{-7}$	$1.7256\ 10^{-7}$	$2.3866\ 10^{-5}$	$2.7655\ 10^{-5}$
	r2+r4	$2.1521\ 10^{-8}$	$2.1701\ 10^{-8}$	$1.6303\ 10^{-7}$	$1.7071\ 10^{-7}$	$2.4254\ 10^{-5}$	$2.8174\ 10^{-5}$
	r3+r4	$2.1777\ 10^{-8}$	$2.2026\ 10^{-8}$	$1.7586\ 10^{-7}$	$1.8326\ 10^{-7}$	$3.2127\ 10^{-5}$	$3.6229\ 10^{-5}$
	poly	**$2.057*10^{-8}$**		**$1.354*10^{-7}$**		**$1.520*10^{-5}$**	
P_e	q	1.3					
	t/T	t/T=0.05		t/T=0.1		t/T=0.2	
α	b	b=0.01	b=0.05	b=0.01	b=0.05	b=0.01	b=0.05
0.25	FE	**$5.812*10^{-8}$**		**$1.298*10^{-6}$**		**$3.568*10^{-4}$**	
	r1+r2+r3	*$5.0121\ 10^{-8}$*	*$4.9832\ 10^{-8}$*	*$9.9596\ 10^{-7}$*	*$1.0228\ 10^{-6}$*	*$2.6251\ 10^{-4}$*	*$2.8150\ 10^{-4}$*
	r2+r3	$5.3701\ 10^{-8}$	$5.2350\ 10^{-8}$	1.116710^{-6}	$1.1187\ 10^{-6}$	$2.9664\ 10^{-4}$	$3.1224\ 10^{-4}$
	r2+r4	$5.4757\ 10^{-8}$	$5.3058\ 10^{-8}$	1.163110^{-6}	$1.1503\ 10^{-6}$	$3.1297\ 10^{-4}$	$3.2349\ 10^{-4}$
	r3+r4	$5.0583\ 10^{-8}$	$5.0118\ 10^{-8}$	1.016310^{-6}	$1.0324\ 10^{-6}$	$2.6982\ 10^{-4}$	$2.8424\ 10^{-4}$
	poly	**$4.734*10^{-8}$**		**$8.834*10^{-7}$**		**$2.241*10^{-4}$**	
0.35	FE	**$3.925*10^{-8}$**		**$5.402*10^{-7}$**		**$1.013*10^{-4}$**	
	r1+r2+r3	*$3.3904\ 10^{-8}$*	*$3.3957\ 10^{-8}$*	*$4.4303\ 10^{-7}$*	*$4.6266\ 10^{-7}$*	*$8.9642\ 10^{-5}$*	*$1.0019\ 10^{-4}$*
	r2+r3	$3.6225\ 10^{-8}$	$3.5637\ 10^{-8}$	4.756910^{-7}	$4.8529\ 10^{-7}$	$9.0018\ 10^{-5}$	$9.9639\ 10^{-5}$
	r2+r4		$3.6256\ 10^{-8}$	4.496810^{-7}	$5.0128\ 10^{-7}$	$9.5204\ 10^{-5}$	$1.0363\ 10^{-4}$
	r3+r4	$3.4275\ 10^{-8}$	$3.4232\ 10^{-8}$	4.53510^{-7}	$4.6938\ 10^{-7}$	$9.2606\ 10^{-5}$	$1.0179\ 10^{-4}$
	poly	**$3.290*10^{-8}$**		**$3.839*10^{-7}$**		**$6.563*10^{-5}$**	
0.5	FE	**$2.413*10^{-8}$**		**$1.858*10^{-7}$**		**$2.088*10^{-5}$**	
	r1+r2+r3	*$2.1533\ 10^{-8}$*	*$2.1770\ 10^{-8}$*	*$1.6765\ 10^{-7}$*	*$1.7543\ 10^{-7}$*	*$2.7706\ 10^{-5}$*	*$3.1765\ 10^{-5}$*
	r2+r3	$2.2498\ 10^{-8}$	$2.2418\ 10^{-8}$	1.718610^{-7}	$1.7761\ 10^{-7}$	$2.2460\ 10^{-5}$	$2.6069\ 10^{-5}$
	r2+r4	$2.3031\ 10^{-8}$	$2.2858\ 10^{-8}$	1.799710^{-7}	$1.8475\ 10^{-7}$	$2.3480\ 10^{-5}$	$2.7004\ 10^{-5}$
	r3+r4		$2.1981\ 10^{-8}$	1.727510^{-7}	$1.7961\ 10^{-7}$	$2.8738\ 10^{-5}$	$3.2521\ 10^{-5}$
	poly	**$2.057*10^{-8}$**		**$1.354*10^{-7}$**		**$1.520*10^{-5}$**	
P_e	q	1.7					
	t/T	t/T=0.05		t/T=0.1		t/T=0.2	
α	b	b=0.01	b=0.05	b=0.01	b=0.05	b=0.01	b=0.05
0.25	FE	**$5.812*10^{-8}$**		**$1.298*10^{-6}$**		**$3.568*10^{-4}$**	
	r1+r2+r3	*$5.0121\ 10^{-8}$*	*$4.9832\ 10^{-8}$*	*$9.9596\ 10^{-7}$*	*$1.0228\ 10^{-6}$*	*$2.6251\ 10^{-4}$*	*$2.8150\ 10^{-4}$*
	r2+r3	$5.5659\ 10^{-8}$	$5.3623\ 10^{-8}$	$1.2002\ 10^{-6}$	$1.1770\ 10^{-6}$	$3.2531\ 10^{-4}$	$3.3342\ 10^{-8}$
	r2+r4	$5.8825\ 10^{-8}$	$5.5698\ 10^{-8}$	$1.3545\ 10^{-6}$	$1.278\ 10^{-6}$	$3.8272\ 10^{-4}$	$3.7118\ 10^{-4}$
	r3+r4	$5.1316\ 10^{-8}$	$5.3623\ 10^{-8}$	$1.0402\ 10^{-6}$	$1.0505\ 10^{-6}$	$2.7640\ 10^{-4}$	$2.8945\ 10^{-4}$
	poly	**$4.734*10^{-8}$**		**$8.834*10^{-7}$**		**$2.241*10^{-4}$**	
0.35	FE	**$3.925*10^{-8}$**		**$5.402*10^{-7}$**		**$1.013*10^{-4}$**	
	r1+r2+r3	*$3.3904\ 10^{-8}$*	*$3.3957\ 10^{-8}$*	*$4.4303\ 10^{-7}$*	*$4.6266\ 10^{-7}$*	*$8.9642\ 10^{-5}$*	*$1.0019\ 10^{-4}$*
	r2+r3	$3.7707\ 10^{-8}$	$3.6686\ 10^{-8}$	$5.1147\ 10^{-7}$	$5.1083\ 10^{-7}$	$9.7965\ 10^{-5}$	$1.0554\ 10^{-4}$
	r2+r4	$4.0281\ 10^{-8}$	$3.8623\ 10^{-8}$	$5.8836\ 10^{-7}$	$5.6826\ 10^{-7}$	$1.1927\ 10^{-4}$	$1.2185\ 10^{-4}$
	r3+r4	$3.4738\ 10^{-8}$	$3.459\ 10^{-8}$	$4.5967\ 10^{-7}$	$4.7359\ 10^{-7}$	$9.2461\ 10^{-5}$	$1.0130\ 10^{-4}$
	poly	**$3.290*10^{-8}$**		**$3.839*10^{-7}$**		**$6.563*10^{-5}$**	
0.5	FE	**$2.413*10^{-8}$**		**$1.858*10^{-7}$**		**$2.088*10^{-5}$**	
	r1+r2+r3	*$2.1533\ 10^{-8}$*	*$2.1770\ 10^{-8}$*	*$1.6765\ 10^{-7}$*	*$1.7543\ 10^{-7}$*	*$2.7706\ 10^{-5}$*	*$3.1765\ 10^{-5}$*
	r2+r3	$2.3416\ 10^{-8}$	$2.3125\ 10^{-8}$	$1.8392\ 10^{-7}$	$1.8757\ 10^{-7}$	$2.3339\ 10^{-5}$	$2.6763\ 10^{-5}$
	r2+r4	$2.5220\ 10^{-8}$	$2.4627\ 10^{-8}$	$2.1512\ 10^{-7}$	$2.1478\ 10^{-7}$	$2.8513\ 10^{-5}$	$3.1585\ 10^{-5}$
	r3+r4	$2.1967\ 10^{-8}$	$2.2102\ 10^{-8}$	$1.7380\ 10^{-7}$	$1.8022\ 10^{-7}$	$2.7535\ 10^{-5}$	$3.1155\ 10^{-5}$
	poly	**$2.057*10^{-8}$**		**$1.354*10^{-7}$**		**$1.520*10^{-5}$**	