A class of improved Nyquist pulses

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Abstract

A novel class of ISI-free pluses is presented. We propose a class of new Nyquist pulses that shows comparable or better ISI performance in the presence of sampling errors, as compared with some recently proposed pulses.
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1 Introduction

The most popular Nyquist pulse [1] is the raised-cosine (RC) pulse, which is produced by a low-pass filter with odd symmetry about the corresponding ideally bandlimited cutoff frequency

$$S_{RC}(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ \frac{1}{2}+\cos\left(\frac{\pi}{2B\alpha}(|f|-B(1-\alpha))\right), & B(1-\alpha)\le|f|\le B(1+\alpha)\\ 0, & B(1+\alpha)<|f|\end{cases} \tag{1}$$

where B is the bandwidth corresponding to symbol repetition rate $T=1/2B$.
The corresponding (scaled) time function is

$$p_{RC}(t)=\sin c(t/T)\frac{\cos(2\pi\alpha t/T)}{1-4\alpha^2t^2/T^2} \tag{2}$$

Recently, improved Nyquist pulses that show smaller maximum distortion, more open receiver eye and a smaller symbol error rate in the presence of symbol timing error were reported [2,3 and 4]. They are defined by

$$S_i(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ G\left(\left(|f|-B(1-\alpha)\right)\right), & B(1-\alpha)|\le |f|| \le B\\ 1-G\left(\left(B(1+\alpha)-|f|\right)\right), & B<|f|\le B(1+\alpha)\\ 0, & B(1+\alpha)<|f|\end{cases} \tag{3}$$

where $G(f)$ is a function satisfying $G(0)=1$. In [2,3 and 4] $G(f)$ was chosen to have a concave shape in the frequency interval $B(1-\alpha)|\le |f|| \le B$ in order to transfer some energy to the high frequency spectral range. This results in a pulse that decays asymptotically as t^{-2} as compared with t^{-3} for the RC pulse, but with the advantage that the eye diagram is more open and, as a consequence, a better bit error rate is obtained.

Two recent contributions showed that improved Nyquist pulses can be obtained with the *flipped-* $G(f)$ technique, e.g. *flipped-exponential* [2] and *flipped-hyperbolic secant* or *flipped-inverse hyperbolic secant* [3].

The envelope of the impulse response decays as t^{-2} or t^{-3} at best, since the functions and their flipped counterparts are continuous at $f_n=1$.

The first derivative of the *flipped-hyperbolic secant* is.continuous at $f_n=1$, which accounts for its steeper decay. The *flipped-exponential* technique uses $G(f)=e^f$ and $\beta=\ln 2/(\alpha B)$, while in [3] $G(f)=\sec h(f)$ and $\beta=\gamma=\ln(\sqrt{3}+2)/(\alpha B)$ or $X(f)=1-\frac{1}{2\alpha B\gamma}arc\sec h(f)$ with $\beta=\frac{1}{2\alpha B}$.

2 A class of new Nyquist pulses

2.1 Frequency- and time domain characteristics

We propose o class of new Nyquist pulses, that are defined for positive frequencies as in (3), with

$$G_i(f)=\frac{1}{2B^i\alpha^i}(B-f)^i+\frac{1}{2} \tag{4}$$

For i odd they show odd symmetry around B and their definition can be

$$S_i(f)=\begin{cases}1, & |f|\le B(1-\alpha)\\ G(f) & B(1-\alpha)|\le |f|| \le B(1+\alpha)\\ 0, & B(1+\alpha)<|f|\end{cases}. \tag{5}$$

For i even, the vestigial symmetry is obtained by choosing $G(f)$ for the frequency interval $B(1-\alpha) \le f \le B$ and $1-G(f)$ for $B \le f \le B(1+\alpha)$. This technique will be denoted in the sequel as *flipped-* $G(f)$ [4].

Figure 1 illustrates this class of new Nyquist filter characteristics for $i = 2, 3, and\ 4$ together with the flipped exponential (FE) defined in [2], taken as a reference.

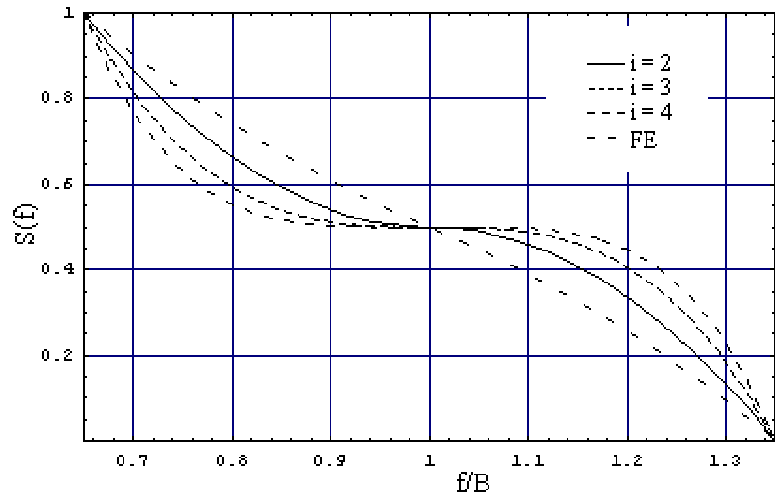

Figure 1 Frequency characteristics of the proposed pulses for an excess bandwidth $\alpha = 0.35$ (positive frequencies only).

Since they are more concave than the FE pulse, a decrease of the first side lobe in time domain is to be expected, as shown in Fig.2.

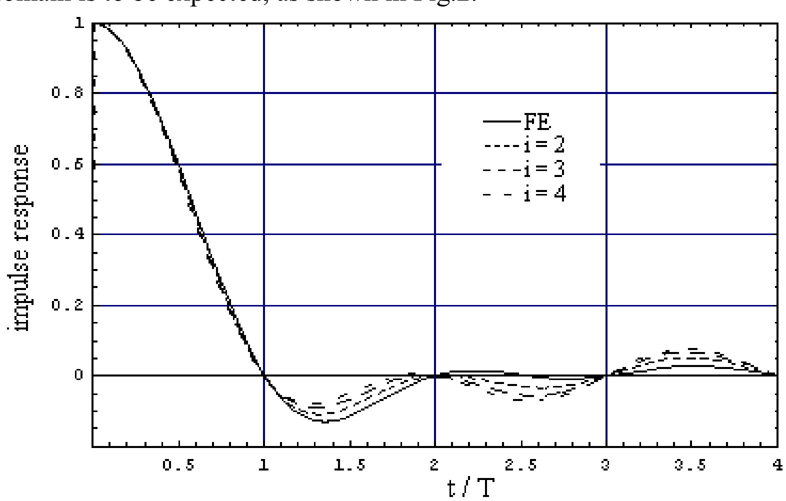

Figure 2 Impulse responses with roll-off factor $\alpha = 0.35$

The impulse responses $s_i(t)$ are given by

$$
\begin{aligned}
s_1(t) &= \frac{\sin(2\pi t)\sin(2\pi\alpha t)}{4\alpha\pi^2 t^2} \\
s_2(t) &= s_1(t)\left(2-\frac{\tan(\pi\alpha t)}{\pi\alpha t}\right) \\
s_3(t) &= s_1(t)\left(3-\frac{3}{2\pi^2\alpha^2 t^2}+\frac{3\cot(2\pi\alpha t)}{\pi\alpha t}\right) \\
s_4(t) &= s_1(t)\frac{4x^3-6x+(6x^2-3)\cot(2x)+3\csc(2x)}{x^3}
\end{aligned}
\tag{6}
$$

where $x=\pi\alpha t$.

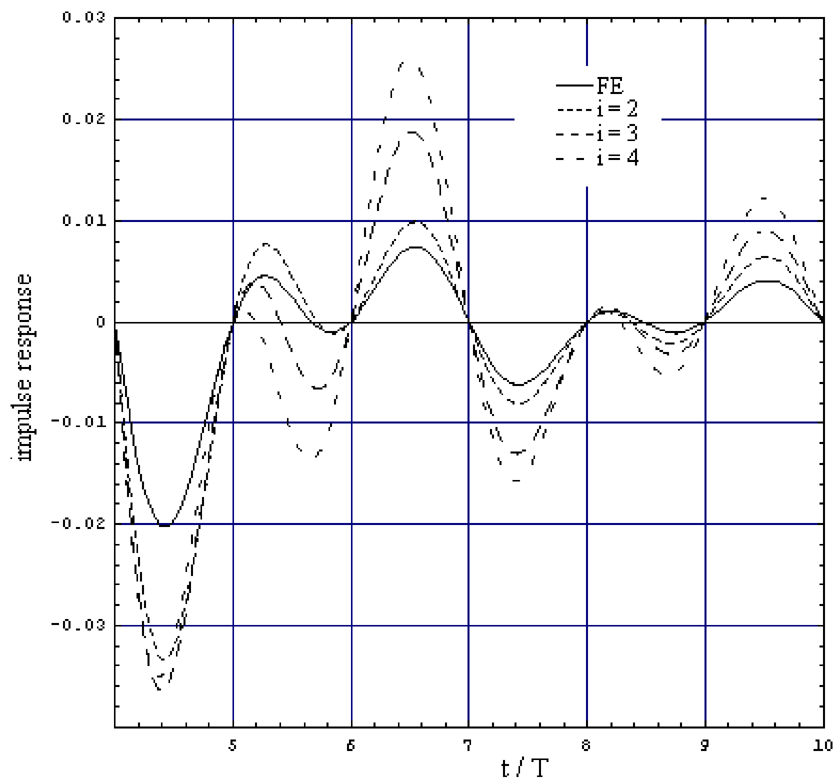

Figure 3 Impulse responses with roll-off factor $\alpha=0.35$

A look at Fig.3 that further illustrates the decay of impulse responses reveals that the new pulse defined by (4) with $i = 2$ or (6) follows closely the FE pulse. Regarding the other pulses ($i = 3$ *and* 4), though the decrease of the first side lobe is more significant, the side lobes are significantly larger, which results in increased ISI. Figure 4 illustrates the receiver eye diagram for FE pulse and the new pulse with i =2.

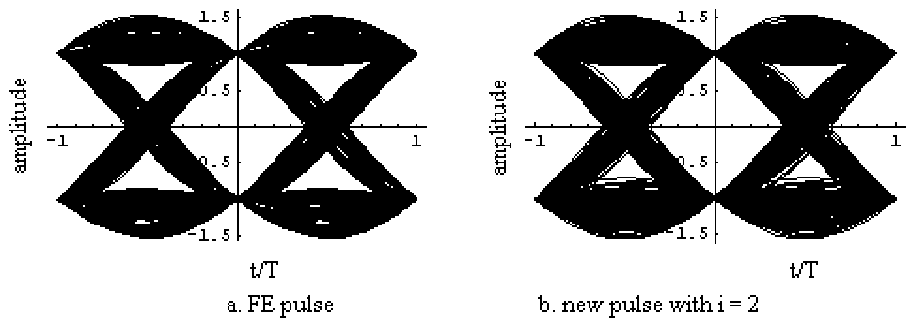

Figure 4 Eye diagram of pulses sequences for an excess bandwidth $\alpha = 0.35$.

2.2 Error probability

When the receiver eye is sampled off center, as in practical receivers, timing error results in an increase of the average symbol error probability [2, 3, 14]. This is calculated using the method of [14] for all proposed pulses and illustrated in Table I, together with those for FE pulse.

P_e	α	$t/T_B = \pm 0.05$	$t/T_B = \pm 0.1$	$t/T_B = \pm 0.2$	$t/T_B = \pm 0.25$
FE	0.25	$5.81166*10^{-8}$	$1.29804*10^{-6}$	$3.56785*10^{-4}$	$2.94623*10^{-3}$
	0.35	$3.92526*10^{-8}$	$5.40211*10^{-7}$	$1.01287*10^{-4}$	$9.35356*10^{-4}$
	0.5	$2.41342*10^{-8}$	$1.85795*10^{-7}$	$2.08778*10^{-5}$	$2.01544*10^{-4}$
	0.75	$1.38358*10^{-8}$	$4.56676*10^{-8}$	$3.22601*10^{-6}$	$4.1329*10^{-5}$
	1	$1.3149*10^{-8}$	$3.56922*10^{-8}$	$1.6144*10^{-6}$	$2.22729*10^{-5}$
S_2 i=2	0.25	$5.29098*10^{-8}$	$1.08226*10^{-6}$	$2.84535*10^{-4}$	$2.3886*10^{-3}$
	0.35	$3.55379*10^{-8}$	$4.59398*10^{-7}$	$8.64181*10^{-5}$	$8.00848*10^{-4}$
	0.5	$2.20606*10^{-8}$	$1.66335*10^{-7}$	$2.22196*10^{-5}$	$2.25693*10^{-4}$
	0.75	$1.44199*10^{-8}$	$5.25581*10^{-8}$	$5.92678*10^{-6}$	$9.30352*10^{-5}$
	1	$1.89674*10^{-8}$	$1.03937*10^{-7}$	$7.97992*10^{-6}$	$7.70296*10^{-5}$
S_3 i=3	0.25	$5.02974*10^{-8}$	$1.00501*10^{-6}$	$2.66064*10^{-4}$	$2.2259*10^{-3}$
	0.35	$3.40523*10^{-8}$	$4.497*10^{-7}$	$9.21857*10^{-5}$	$8.48554*10^{-4}$
	0.5	$2.17085*10^{-8}$	$1.72143*10^{-7}$	$2.93619*10^{-5}$	$3.24192*10^{-4}$
	0.75	$1.71828*10^{-8}$	$8.82974*10^{-8}$	$1.26322*10^{-5}$	$1.94043*10^{-4}$

	1	$2.98321*10^{-8}$	$3.26661*10^{-7}$	$5.24233*10^{-5}$	$4.41245*10^{-4}$
s_4 $i=4$	0.25	$4.934224*10^{-8}$	$9.92992*10^{-7}$	$2.68781*10^{-4}$	$2.22926*10^{-3}$
	0.35	$3.37477*10^{-8}$	$4.64803*10^{-7}$	$1.03168*10^{-4}$	$9.41583*10^{-4}$
	0.5	$2.20729*10^{-8}$	$1.85251*10^{-7}$	$3.80746*10^{-5}$	$4.43739*10^{-4}$
	0.75	$2.03095*10^{-8}$	$1.41921*10^{-7}$	$2.32649*10^{-5}$	$3.21185*10^{-4}$
	1	$4.15202*10^{-8}$	$7.05569*10^{-7}$	$1.68712*10^{-4}$	$1.34152*10^{-3}$

Table 1: ISI error probability of several Nyquist pulses for $N=2^{10}$ interfering symbols and SNR = 15 dB

3 Conclusions

A new class of Nyquist pulses that show reduced maximum distortion, a more open receiver eye and decreased symbol error probability in the presence of timing error as compared with the FE pulse [2] with the same roll-off factor was introduced. Its transmission properties were thoroughly investigated and show that the pulses have practical importance.

References

[1] Nyquist, H. (1928) "Certain topics in telegraph transmission theory" *AIEE Trans.*, vol. 47, pp. 617–644.

[2] Beaulieu, N. C., Tan, C. C., and Damen, M. O. (2001) "A "better than" Nyquist pulse," *IEEE Commun. Lett.*, vol. 5, pp. 367–368, Sept..

[3] Assalini, A.and Tonello, A. M. (2004) "Improved Nyquist pulses," *IEEE Communications Letters,* vol. 8, pp. 87 - 89, February.

[4] Beaulieu N. C.and Damen, M. O., (2004) "Parametric construction of Nyquist-I pulses," *IEEE Trans. Communications,* vol. COM-52, pp. 2134 - 2142, December.

[5] Lucky, R. W., Salz, J,.and Weldon Jr., E. J. (1968) "Principles of Data Communication". New York: McGraw-Hill,.

[6] Xia, X.-G. (1997) "A family of pulse-shaping filters with ISI-free matched and unmatched filter properties," *IEEE Trans. Commun.*, vol. 45, pp. 1157–1158, Oct..

[7] Demeechai, T. (1998) "Pulse-shaping filters with ISI-free matched and unmatched filter properties," *IEEE Trans. Commun.*, vol. 46, p. 992, Aug..

[8] Tan, C. C. and Beaulieu, N. C. (2004) "Transmission properties of conjugate-root pulses," *IEEE Trans. Commun.*, vol. 52, pp. 553–558, Apr.

[9] Franks, L. E. (1968) "Further results on Nyquist's problem in pulse transmission," *IEEE Trans. Commun. Technol.*, vol. COM-16, pp. 337–

340, Apr.
[10] Kisel A. V., (1999) "An extension of pulse shaping filters theory," *IEEE Trans. Commun.*, vol. 47, pp. 645–647, May.
[11] (2000) "Nyquist 1 universal filters," *IEEE Trans. Commun.*, vol. 48, pp.1095–1099, July.
[12] Andrisano, O. and Chiani, M. (1994) "The first Nyquist criterion applied to coherent receiver design for generalized MSK signals," *IEEE Trans. Commun.*, vol. 42, Feb./Mar./Apr.
[13] Sayar, B. andPasupathy, S. (1987). "Nyquist 3 pulse shaping in continuous phase modulation," *IEEE Trans. Commun.*, vol. COM-35, pp. 57–67, Jan.
[14] Beaulieu, N. C. (1991) "The evaluation of error probabilities for intersymbol and cochannel interference," *IEEE Trans. Commun.*, vol. 31, pp.1740–1749, Dec.
[15] Hill Jr., F. S. (1977) "A unified approach to pulse design in data transmission," *IEEE Trans. Commun.*, vol. COM-25, pp. 346–354, Mar..
[16] Alagh, N. S. and Kabal, P. (1999) "Generalized raised-cosine filters," *IEEE Trans. Commun.*, vol. 47, pp. 989–997, July.